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DEPARTMENT OF AERONAUTICAL ENGINEERING

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Preliminary Report on the Theoretical Treatment of the Two-dimensional Ground Effect Machine in Forward Motion.

by

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TABLE OF CONTENTS

I. INTRODUCTION

The latter half of this decade has seen a rapid development of the ideas first incorporated by Kaario in his ram wing in 1935 and his hovercraft in 1949. Considerable theoretical and experimental efforts have been applied to the understanding of the basic principles involved in, what has become known as, the G.E.M. ie., a machine supported on a cushion of compressed air sustained and contained by a peripheral jet. It may be said that the theory of the momentum curtain is fairly well understood. Such an understanding has come about through the work of Chaplin (1) via various refinements to the exact perfect, incompressible fluid theory of Strand (2) for the two-dimensional model. However, although well understood, the theory is not, of course, complete. Amongst the outstanding problems is that of the translational motion of the G.E.M. The object of this paper is to partially fulfill this requirement.

Throughout this study the only information available to the author on the effects of forward motion on the G.E.M. was that contained in the Symposium on Ground Effect Phenomena (3) publication, Princeton University. In these reports the models used were various - small wind tunnel models and large scale vehicles were used; some models having thin, high,

velocity jets others having thick low velocity jets; some with simulated or actual intake, others without. Consistent with the variation in configuration has been the inconsistency of reports on the effect of forward flight on the base pressure (see Ref. 3 p. 309). However in all cases included in reference 3 the variation in base pressure was reported as being small for low forward speeds. In order to define distinct regimes of operation it will be assumed that the base pressure remains invariant with forward speed provided that the jet at the leading edge escapes upstream on contact with the ground. Such a regime will correspond to high values of the non-dimensional jet momentum coefficient, C_J, defined as the ratio of the jet momentum efflux to the free stream dynamic pressures times a representative length (diameter of the machine). If these arguments are accepted the problem of the G.E.M. in translation becomes one of a semi-infinite uniform stream flowing over an obstacle on the ground. It is as such that the mathematical model of the problem is formulated; that is to say, a solution is sought to the problem of the uniform flow of a perfect fluid external to the G.E.M.

II. DISCUSSION

A. Mathematical Model

The problem is restricted to a two-dimensional configuration shown in Figure 1 where AB, EF represent the ground, BC, ED the jet profiles and CD the flat plate representation of the machine. It is required to solve the problem of the uniform, irrotational flow of a perfect, incompressible fluid of speed U over the boundary A B C D E F.

B. Method of Solution

The complex potential plane drawn in Figure 2 is mapped onto a semi-infinite rectangle in the ζ-plane in Figure 3 by means of the Schwartz-Christoffel transformation, given by

$$W = \frac{\phi_1 + \phi_2}{2} \cosh \frac{\zeta}{2} - \frac{\phi_1 + \phi_2}{2}$$

The development of a theorem due to Woods (4) is repeated below with but a slight adaptation to the problem of this paper. By applying Cauchy's Integral Theorem to the function

$$f(\zeta) = \log \frac{U}{q} + i\theta ,$$

where q is the speed and θ the direction of motion and ζ the complex coordinate of point P within the principle semi-infinite rectangle, A*B*E*F*, of the transformation, it is found that, as the limit of $f(\zeta)$ as $\zeta \to \infty$ is zero,

$$-f(\zeta) = \frac{1}{2\pi i}\left[\int_0^{2\pi}\frac{f(0,\gamma)\,i\,d\gamma}{i\gamma-\zeta}\right.$$

$$\left.+\int_0^{\infty}\left\{\frac{f(\eta,2\pi)}{\eta+i2\pi-\zeta} - \frac{f(\eta,0)}{\eta-\zeta}\right\}d\eta\right]; \qquad (1)$$

and for every other semi-infinite rectangle A B E F

$$0 = \frac{1}{2\pi i}\left[\int_{2m\pi}^{\overline{2m+2\pi}}\frac{f(0,\gamma)\,i\,d\gamma}{i\gamma-\zeta}\right.$$

$$\left.+\int_0^{\infty}\left\{\frac{f(\eta,\overline{2m+2\pi})}{\eta+i\overline{2m+2\pi}-\zeta} - \frac{f(\eta,2m\pi)}{\eta+i2m\pi-\zeta}\right\}d\eta\right] \qquad (2)$$

On substituting $\gamma+2m\pi$ for γ in the first term, equation (2) becomes

$$0 = \frac{1}{2\pi i}\left[\int_0^{2\pi}\frac{f(0,\gamma+2m\pi)\,i\,d\gamma}{i\gamma+i2\pi m\pi-\zeta}\right.$$

$$\left.+\int_0^{\infty}\left\{\frac{f(\eta,\overline{2m+2\pi})}{\eta+i\overline{2m+2\pi}-\zeta} - \frac{f(\eta,2m\pi)}{\eta+i2m\pi-\zeta}\right\}d\eta\right]$$

Hence we can write

$$\left.\begin{array}{ll} m=0 & -f(\zeta) \\ m\neq 0 & 0 \end{array}\right\} = \frac{1}{2\pi i}\left[\int_0^{2\pi}\frac{f(0,\gamma+2m\pi)\,i\,d\gamma}{i\gamma+i2m\pi-\zeta}\right.$$

$$\left.+\int_0^{\infty}\left\{\frac{f(\eta,\overline{2m+2\pi})}{\eta+i\overline{2m+2\pi}-\zeta} - \frac{f(\eta,2m\pi)}{\eta+i2m\pi-\zeta}\right\}d\eta\right] \qquad (3)$$

It is to be noticed that in these equations the sides AB and EF are covered twice but in opposite directions i.e.

$$\int_0^\infty \frac{f(\eta, \overline{2m+2\pi})d\eta}{\eta + i2m + 2\pi - \zeta} \qquad \text{when } m = n$$

is equal to

$$\int_0^\infty \frac{f(\eta, 2m\pi)d\eta}{\eta + i2m + 2\pi - \zeta} \qquad \text{when } m = n + 1$$

Also, in the limit as $m \to \infty$ these integrals tend to zero. Hence, if equations (3) are summed from $m = -\infty$ to $m = +\infty$ the function $f(\zeta)$ is found to be given by

$$-f(\zeta) = \frac{1}{2\pi i} \int_0^{2\pi} \sum_{m=-\infty}^{+\infty} \frac{f(0, \gamma + 2m\pi)}{i\gamma + i2m\pi - \zeta} \, i d\gamma$$

The summation in the above equation can be split into two parts; the first being a summation over the range of even values of m and the second over the range of odd values of m. By writing $m = 2n$ and $2n + 1$ respectively we get

$$-f(\zeta) = \frac{1}{2\pi i} \left[\int_0^{2\pi} \sum_{n=-\infty}^{\infty} \frac{f(0, \gamma + 4n\pi)}{i\gamma + i4n\pi - \zeta} i d\gamma + \int_0^{2\pi} \sum_{n=-\infty}^{\infty} \frac{f(0, \gamma + \overline{4n+2\pi})}{i\gamma + i4n + 2\pi - \zeta} i d\gamma \right] \tag{4}$$

But when m is odd

$$f(0, 4m\pi - \gamma) = f(0, \gamma)$$

thus, substituting $(4m\pi - \gamma)$ for $(\gamma + 2m\pi)$ in the second integral of (4), and remembering that $f(0, \gamma + 2m\pi)$ is also equal to $f(0, \gamma)$ for even values of m in the first integral, the equation becomes

$$-f(\zeta) = \frac{1}{2\pi i}\left[\int_0^{2\pi} f(0,\gamma)\left\{\sum_{n=-\infty}^{\infty} \frac{1}{i\gamma + i4n\pi - \zeta} + \sum_{n=-\infty}^{\infty} \frac{1}{-i\gamma + i4n\pi + i4\pi - \zeta}\right.\right]$$

$$= \frac{1}{8\pi}\left[\int_0^{2\pi} f(0,\gamma)\left\{\coth\frac{1}{4}(i\gamma - \zeta) - \coth\frac{1}{4}(i\gamma + \zeta)\right\} d\gamma\right] \qquad (5)$$

on using the identity

$$\lim_{M\to\infty} \sum_{n=-M}^{+M} (a+bn)^{-1} = \frac{\pi}{b}\cot\frac{\pi a}{b}$$

In equation (5) for ζ write $-\bar{\zeta}$. Then, as the point $-\bar{\zeta}$ is outside all rectangles we have

$$0 = \frac{1}{8\pi}\left[\int_0^{2\pi} f(0,\gamma)\left\{\coth\frac{1}{4}(i\gamma + \bar{\zeta}) - \coth\frac{1}{4}(i\gamma - \bar{\zeta})\right\} d\gamma\right]$$

the conjungate of which gives

$$0 = \frac{1}{8\pi}\left[\int_0^{2\pi} \bar{f}(0,\gamma)\left\{\coth\frac{1}{4}(i\gamma - \zeta) - \coth\frac{1}{4}(i\gamma + \zeta)\right\} d\gamma\right] \qquad (6)$$

Now $\quad f(\zeta) = \log\frac{U}{q} + i\theta$

so that $\bar{f}(\zeta) = \log\frac{U}{q} - i\theta$

Hence, on substracting equation (6) from (5) we finally obtain

$$-f(\zeta) = \frac{1}{4\pi}\int_0^{2\pi} \theta(\gamma)\left\{\cot\frac{1}{4}(\gamma + i\zeta) - \cot\frac{1}{4}(\gamma - i\zeta)\right\} d\gamma \qquad (7)$$

C. Approximations

Two approximations can be made which the author thinks are justified for this simple model. It may be assumed that the flow over the boundary BCDE (Fig. 1) is sufficiently symmetrical about the mid point, $\gamma = \pi$, to justify writing

$$\theta(\gamma) = \theta(2\pi - \gamma)$$

In which case equation (7) reduces to

$$-f(\zeta) = \frac{1}{2\pi}\int_0^{2\pi} \theta(\gamma^*) \cot \tfrac{1}{2}(\gamma^* + i\zeta)\, d\gamma^* \tag{8}$$

where γ has been replaced by γ^* in order to differentiate between the dummy variable and the imaginary part of ζ . On the ground effect machine boundary i.e. the boundary made up of the jet profiles and the upper surface of the G.E.M., $\eta = 0$ so that $\zeta = i\gamma$ and the right hand side of (8) is real. Hence, the left hand side becomes $-\log U/q$ and is given by

$$-\log \frac{U}{q} = \frac{1}{2\pi}\int_0^{2\pi} \theta(\gamma^*) \cot \tfrac{1}{2}(\gamma^* - \gamma)\, d\gamma^* \tag{9}$$

which can be transformed into a Stieltjes Integral by integration by parts to give

$$+\log \frac{U}{q} - \frac{1}{\pi}\int_0^{2\pi} \log \sin \tfrac{1}{2}(\gamma^* - \gamma)\, d\theta(\gamma^*) \tag{10}$$

The evaluation of this integral leads us to the second approximation.

If the jet is thick and the forward speed, U, is small the jet momentum coefficient will be large and the jet will be practically straight up to the ground where the change of curvature will be rapid. This is also true for a thin jet but, in this case, the range of U will be increased within the validity of the assumption (5). Thus, the curvillinear boundaries BC and ED will be replaced by the rectilinear boundaries BB'C and EE'D respectively (see Fig. 1) to give the $\theta - \gamma$ distribution shown in Fig. 4. The evaluation of the Stieltjes Integral in equation (10) then gives

$$\log \frac{U}{q} = \frac{1}{\pi}\Big[\left(\frac{\pi}{2}+\tau\right)\log \sin\tfrac{1}{2}\left|(\epsilon-\gamma)\right| - \left(\frac{\pi}{2}+\tau\right)\log\sin\tfrac{1}{2}\left|(\delta-\gamma)\right|$$

$$-(\pi-\tau)\log\sin\tfrac{1}{2}\left|(2\pi-\delta-\gamma)\right|$$

$$+(\pi-\tau)\log\sin\tfrac{1}{2}\left|(2\pi-\epsilon-\gamma)\right|\Big] \qquad (11)$$

Thus the distribution of q is known as a function of γ . From equation (8), where $f(\zeta) \equiv \log\left(-U/\frac{dW}{dz}\right)$, dW/dz may be calculated as a function of γ . This, together with W obtained as a function of γ from equation (1), which, in the symmetrical case becomes

$$W = \phi_1 \cosh\frac{\zeta}{2} \qquad (12)$$

leads to the evaluation of the complex coordinate, z, in the physical plane, as a function of γ. Thus the velocity distribution over the upper surface of the GEM can be obtained. Hence the aerodynamic lift due to the flow over the upper surface of the GEM can be calculated.

The constants ϵ, δ and ϕ_1 may be evaluated from the following relationships:

(1) At B' where $z = z_1$ is known, $\zeta = i\epsilon$

(2) At C where $z = z_2$ is known, $\zeta = i\delta$

(3) $$2\phi_1 = \int_B^E d\phi$$

$$= \int_{BCDE} q\,dz$$

$$= \int_{2\pi}^{0} q(\gamma)\frac{dz}{d\gamma}\,d\gamma .$$

Thus, it is seen that equations (11), (8) and (12) completely solve the problem as set out above.

III. CONCLUDING REMARKS

The above method due to L. C. Woods (4) has been applied by him to many problems where refinements of computation are described in detail. The author wishes to stress the preliminary nature of this report which he would wish to be regarded as an introduction to the more complete non-symmetrical problem of a two-dimensional ground effect machine in forward motion with a simulated intake on the upper surface and jet exits situated on the underside other than at the edges. Work is now in progress on this extended model.

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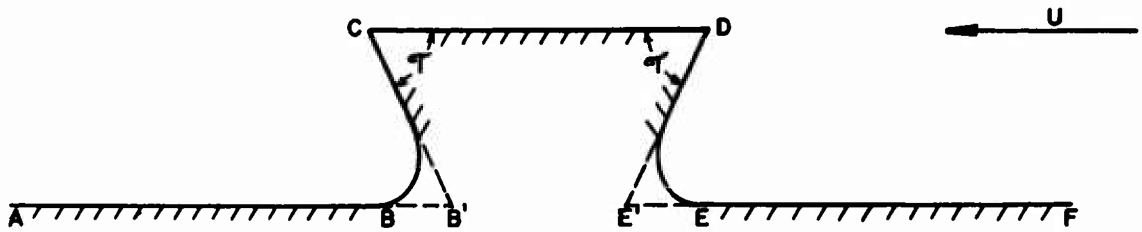

Figure 1. Z - Plane

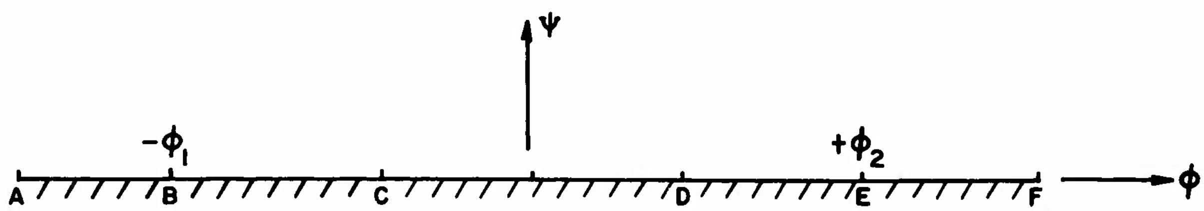

Figure 2. W (= φ + iψ) - Plane

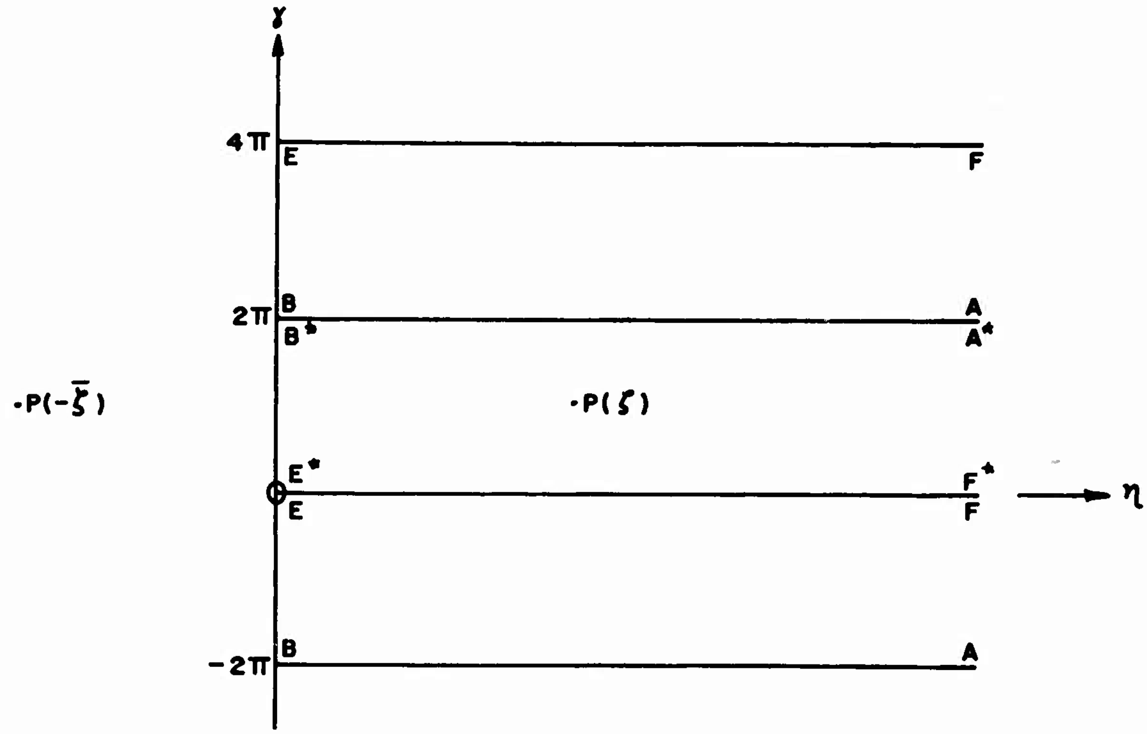

Figure 3. $\zeta(=\eta+i\gamma)$ - Plane

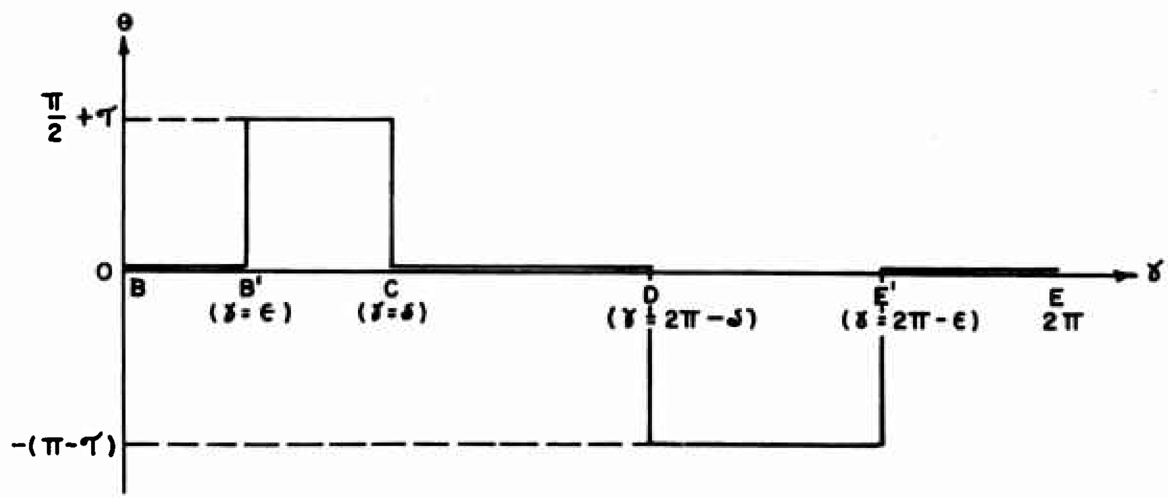

Figure 4. θ - γ Distribution

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